STRUCTURAL AND MECHANICAL PROPERTIES AND FILTRATION IN ELASTIC FISSURED AND POROUS MATERIAL

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Within the framework of the model of two coexisting homogeneous porous continua the article examines the influence of the state of stress of a material and of the liquid pressure on the effective parameters of the continua, and a system of equations of filtration is obtained that differs from the known system.

By fissured and porous material we mean material in which there is a developed system of interconnected cracks separating blocks of porous medium. The notion of such material as two homogeneous porous media, one inserted into the other and each having its porosity and permeability, and the corresponding approach to describing filtration processes in it were first suggested in [1, 2] and explained in detail in [3].

On the basis of a more detailed analysis of the dependence of the structural and mechanical characteristics of fissured and porous material on the state of stress in its phases, the present work, using an analogous continual approach, obtains a system of equations of filtration in the mentioned porous continua that differs substantially from the system of equations obtained in [1-3].

For the sake of simplification, the porous medium forming the blocks is assumed to be homogeneous and isotropic. We consider deformations of the material to be elastic, we completely neglect irreversible plastic deformations, and filtration in the material is taken to be linear. The dependence of density and viscosity of the liquid on the pressure and the dependence of the porosity of the blocks on the stresses in them are assumed to be weak, and fissure porosity is assumed to be small compared with unity.

State of Stress of the Material and Effective Properties of Porous Continua. First of all we will examine the different types of stress acting in fissured and porous material filled with liquid. Representing it as composite material consisting of a porous matrix and cracks distributed in it, we obtain for the full mean stresses

$$\sigma_{ij} = (1 - m_1)\sigma_{ij}^{(1)} + m_1 p_1 \delta_{ij}, \tag{1}$$

where $\sigma_{ij}^{(1)}$ are the mean true (i.e., referred to the fraction of the cross-sectional area required for the porous blocks) stresses in the porous matrix, and the plus sign is assigned to compressive normal stresses. The state of a single crack is determined by the tensor $\sigma_{ij} - p_1\delta_{ij}$ [4]. Bearing in mind that the mean stress in the porous matrix calculated per unit area of the total cross section is $\sigma_{ij}^{(1)\prime} = (1 - m_1)\sigma_{ij}^{(1)}$, and also taking relation (1) into consideration, we see that this tensor coincides with the tensor of the usually introduced fictitious stresses $\sigma_{ij}^{(1)\prime} - (1 - m_1)p_1\delta_{ij}$ for a porous body whose porosity is due solely to cracks [3].

Analogously, the magnitudes $\sigma_{ij}^{(1)}$ playing the part of full mean stresses in the porous blocks can be represented in the form of linear combinations of the mean true stresses in the hard skeleton frame of the blocks and of the normal stresses in the pores, i.e.,

$$\sigma_{ij}^{(1)} = (1 - m_2)\sigma_{ij}^{(2)} + m_2 p_2 \delta_{ij}. \tag{2}$$

The mean stresses in the skeleton frame per total cross-sectional area of the porous block are: $\sigma_{ij}^{(2)\prime} = (1 - m_2)\sigma_{ij}^{(2)}$. Therefore for the corresponding fictitious stresses in the blocks, on which only their porosity and permeability have to depend, we obtain from (1) and (2)

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$$\sigma_{ij}^f = \sigma_{ij}^{(2)\prime} - (1-m_2)\,p_2\delta_{ij} = (1-m_2)(\sigma_{ij}^{(2)} - p_2\delta_{ij}) = \sigma_{ij}^{(1)} - p_2\delta_{ij} = (1-m_1)^{-1}(\sigma_{ij} - m_1 p_1\delta_{ij}) - p_2\delta_{ij} \approx \sigma_{ij} - p_2\delta_{ij}. \quad (3)$$

The last approximate equality in (3) corresponds to the previously expressed assumption that $m_1 \ll 1$. Taking it that the full stresses σ_{ij} are not dependent on time (the conditions necessary for this in connection with real strata were discussed in [3]), we obtain from (3)

$$\frac{\partial \sigma_{ij}^f}{\partial t} = -\frac{\partial p_2}{\partial t}\delta_{ij}, \quad \frac{\partial \sigma^f}{\partial t} = -\frac{\partial p_2}{\partial t}, \quad (4)$$

where σ^f is the first invariant of the fictitious stress tensor (3). Considering ourselves limited by the traditional notions, we neglect here the possible dependence of porosity and permeability of the blocks on other invariants of the tensor σ_{ij}^f, i.e., we adopt

$$m_2 = m_2(p_2, \sigma^f), \quad k_2 = k_2(p_2, \sigma^f). \quad (5)$$

We note that the authors of [1-3] did not examine the dependence of these magnitudes on σ^f, on the other hand they considered them dependent on the pressure in the fissures; this does not seem very consistent.

We will discuss briefly the problem of determining the effective permeability tensors k_1 and k_2 associated with the introduced porous continua in which filtration is expected to model the real averaged motion of liquid through fissures and the porous blocks, respectively. It follows from numerous investigations of the effective properties (thermal conductivity, electromagnetic characteristics, viscosity, moduli of elasticity, etc.) of disperse media and composite materials that the properties of each of the introduced continua depend in a complex manner on the corresponding physical characteristics of all the phases or components of which the system consists, and also on the type of internal structure of the system. This problem is therefore exceedingly complex. For the case that flow through both continua is considerable, there does not exist so far any acceptable solution, not even one based on the physically justified simplifying assumptions. It is only clear that the tensor k_1 in the general case has to depend not only on the topological properties of the system of fissures and on the mean characteristics of one fissure but also on the scalar pearmeability of the porous blocks k_2, and the tensor k_2 has to depend not only on k_2 but also on the properties of the system of fissures. For instance, when there is anisotropy in this system (the tensor k_1 is nonspherical), tensor k_2, generally speaking, will not be spherical either.

However, when the fissure porosity of the blocks is small and their permeability is relatively low, it is apparently permissible to adopt as a fairly reasonable approximation that the tensor k_1 does not differ from the corresponding tensor in material with the same system of fissures but with impermeable blocks, and that tensor k_2 is spherical with unique eigenvalue equal to k_2. For lack of a better model we will examine below this simple approximation.

Since $m_1 \ll 1$, a single fissure may be regarded approximately as being in an unbounded homogeneous elastic medium whose effective modulus of elasticity and Poisson ratio are approximately equal to the analogous magnitudes E and ε of a porous matrix filled with liquid, and in which the stresses σ_{ij} act (at a distance from the fissure). As representative model fissure we regard an oblate ellipsoid of revolution with the semiaxes c and h. On the basis of the results of [4] we have for the opening of the crack

$$h = \frac{4}{\pi} c \frac{(1-\varepsilon^2)(p_1 - \mathbf{n}\cdot\sigma\cdot\mathbf{n})}{E} Y(p_1 - \mathbf{n}\cdot\sigma\cdot\mathbf{n}), \quad (6)$$

where Y(x) is the Heaviside function, and $\mathbf{n}$ is the unit vector of the normal to the plane of the fissure. The volume of the fissure is proportional to c^2h, and its hydraulic conductivity to h^3. Hence follows that the fissure porosity and the components of the tensor of fissure permeability depends very strongly both on the liquid pressure in the fissures and on the state of stress of the material. In particular it is obvious that the effective fissure permeability of the material, which in the unloaded state was isotropic from the macroscopic point, will be described in the loaded state, generally speaking, by a tensor of rank 2 whose principal axes coincide with the principal axes of the stress tensor.

The components of k_1 with arbitrary state of stress of the material and known distribution function of the fissures according to orientations can be calculated in the following manner. In the system there acts the pressure gradient $\nabla p_1 = -\Sigma \alpha_i e_i$, where e_i is the unit vector. In accordance with the Boussinesq—Poiseuille law for a single fissure with specified orientation n, the component of the liquid flow in this fissure, due to the i-th component

∇p_1, is proportional to the vector $h^3(n \times a_i e_i) \times n$, where h is expressed by formula (6). If we average this vector over the directions n using the mentioned distribution function, we obtain as a result some new vector $a_i \Sigma C_{ij} e_j$; all its three components are nonzero in the general case, and the coefficients C_{ij} are functions of p_1 and σ_{kl}. It is clear that with an accuracy of up to the scalar factor these coefficients coincide with the corresponding components of the tensor of fissure permeability, i.e., $k_{1,ij} = AC_{ij}$. The constant A can be most simply determined by using some reference value of permeability. The calculations can be easily generalized to the situation when the fissures are distributed not only according to orientation n, but also according to the dimension c. In the special case, when the unloaded material is macroscopically isotropic, it is expedient to change to the system of coordinates connected with the principal axes of the tensors σ and k_1, and to determine the eigenvalues of the tensor k_1.

It is obvious that if p_1 is larger than the maximum eigenvalue of the tensor σ, all the fissures are open, though the degree of their opening is unequal. If p_1 is smaller than the maximum eigenvalue but larger than the minimum eigenvalue of σ, fissures with a certain orientation are closed and do not take part in the throughflow. If p_1 is smaller than the minimum eigenvalue of σ, then all cracks are closed, i.e., fissure permeability vanishes altogether. Therefore fissure permeability can be substantial only with fairly high liquid pressures within the fissures. This explains to a certain extent the known fact that in deep-lying oil strata, regarded as fissured or fissured-porous, anomalously high rock pressures, much higher than the calculated ones, were always found. In fact, with low rock pressures the fissure permeability in accordance with the theory submitted here simply could not be noticed because the fissures are closed. This may also explain the consistent underestimation of the elastic oil reserved in such strata when standard estimating methods are used. As a rule, the cumulative yield is much larger than the preliminarily calculated reserves.

Thus the dependence of fissure permeability on the state of stress of the material and on the liquid pressure in fissures, even for elastic material, is much more complex, and what is most important, stronger than was usually assumed (see, e.g., [5]). Although the calculation of this dependence is simple in principle, it is very cumbersome. To explain the principle of the matter concerned, we will examine below, as a fairly simple but characteristic illustration, filtration in material exposed to hydrostatic pressure with stress σ. Then for fissure porosity and fissure permeability we have, taking (6) into account,

$$m_1 = m_1^\circ \frac{p_1 - \sigma}{p^\circ - \sigma} Y(p_1 - \sigma), \quad k_1 = k_1^\circ \left(\frac{p_1 - \sigma}{p^\circ - \sigma}\right)^3 Y(p_1 - \sigma). \tag{7}$$

Formulas (7) also describe the properties of material all of whose cracks are oriented in one plane; in that case σ means compressive stress normal to this plane.

Equations of Filtration. We write the equations of the balance of liquid in the fissures and in the porous blocks taking mass exchange between them into account which play the part of equations of the conservation of mass for the introduced porous continua, and also the corresponding Darcy equations

$$\frac{\partial (m_i \rho)}{\partial t} + \nabla \cdot (\rho \mathbf{u}_i) = q_i, \quad \mathbf{u}_i = -\frac{1}{\mu_i} \mathbf{k}_i \cdot \nabla p_i, \tag{8}$$

$$q_1 = -q_2 = q, \quad i = 1, 2.$$

Since the viscosity and density of a liquid depend weakly on the pressure, we put $\mu_i = \mu^\circ$, and for the flow q per unit volume of material we use an expression that was suggested in [1-3]

$$q = \alpha \frac{\rho^\circ k_2^\circ}{\mu^\circ l^2} (p_2 - p_1). \tag{9}$$

Taking (4) and (5) into account, we obtain by the standard method [3] the following equation of elastic filtration regime in porous blocks:

$$m_2^\circ \left(\frac{1}{K_\rho} + \frac{1}{K_m}\right) \frac{\partial p_2}{\partial t} = \frac{k_2^\circ}{\mu^\circ} \Delta p_2 - q, \quad \frac{\rho^\circ}{K_\rho} = \left(\frac{\partial \rho}{\partial p}\right)^\circ, \quad \frac{m_2^\circ}{K_m} = \left(\frac{\partial m_2}{\partial p_2} - \frac{\partial m_2}{\partial \sigma^f}\right)^\circ. \tag{10}$$

In deriving an analogous equation for filtration in fissures, the dependence of ρ on p may be neglected in view of the much stronger dependence of m_1 on p. As a result we obtain from (7) and (8)

$$m_1^\circ \frac{\partial}{\partial t}\left(\frac{p_1-\sigma}{p^\circ-\sigma}\right)=\frac{1}{\mu^\circ}\nabla\cdot\left[\left(\frac{p_1-\sigma}{p^\circ-\sigma}\right)^3 \mathbf{k}_1^\circ\cdot\nabla p_1\right]+q, \tag{11}$$

and this equation has meaning only when $p_1 > \sigma$.

Taking (9) into account and introducing the parameters

$$s=\frac{m_1^\circ}{m_2^\circ}\left(\frac{1}{K_\rho}+\frac{1}{K_m}\right)^{-1},\quad \gamma=\frac{\mathbf{k}_1^\circ}{k_2^\circ}\varkappa, \tag{12}$$

$$\tau=\frac{l^2}{\alpha\varkappa},\quad \varkappa=\frac{k_2^\circ}{m_2^\circ\mu^\circ}\left(\frac{1}{K_\rho}+\frac{1}{K_m}\right)^{-1},$$

we obtain from (10) and (11) the system of equations

$$s\frac{\partial}{\partial t}\left(\frac{p_1-\sigma}{p^\circ-\sigma}\right)=\nabla\cdot\left[\left(\frac{p_1-\sigma}{p^\circ-\sigma}\right)^3\gamma\cdot\nabla p_1\right]+\frac{p_2-p_1}{\tau}, \tag{13}$$

$$\frac{\partial p_2}{\partial t}=\varkappa\Delta p_2-\frac{p_2-p_1}{\tau}.$$

If the characteristic changes of the pressures p_1 and p_2 are sufficiently small in comparison with $p^\circ-\sigma$, and the tensor γ is spherical, then we have from (13) approximately

$$s\frac{\partial p_1}{\partial t}=\gamma\Delta p_1+\frac{p_2-p_1}{\tau}, \tag{14}$$

$$\frac{\partial p_2}{\partial t}=\varkappa\Delta p_2-\frac{p_2-p_1}{\tau}$$

(the constants s and γ in (14) are different from the constants in (13)).

Since permeability through blocks is usually much smaller than through fissures, i.e., $\varkappa\ll\gamma$, we may neglect the term with Δp_2 in the second equation of (14). But this, generally speaking, cannot be done in the corresponding equation of (13) because with p_1 tending to σ, fissure permeability becomes comparable with permeability through blocks (if, however, we are not dealing with fissured material with impermeable blocks), and when $p_1 < \sigma$, it vanishes altogether. In that case, instead of (13) we have an ordinary equation of elastic filtration regime only through blocks with the coefficient of piezoconductivity $\varkappa$. We note that the term with the derivative with respect to time in the first equation of (13) or (14) may not be neglected either in the general case because ususally $K_\rho \gg 1$ and $K_m \gg 1$, and the coefficient s, determined in accordance with (12), may be an order or more larger than unity even when $m_1^\circ \ll m_2^\circ$.

Let us compare (14) with the analogous system suggested in [1-3]. Firstly, in the latter equations there is no term with $s\partial p_1/\partial t$ which, as was already shown, must not be neglected. Secondly, on the left-hand side of the equation for mean pressure in porous blocks [1-3] has instead of $\partial p_2/\partial t$ the magnitude $\partial p_2/\partial t - \beta\partial p_1/\partial t$, where β is a coefficient depending on the properties of the material and of the liquid. This is due to the fact that in [1-3] it is postulated that instead of (5), the dependence of m_2 and k_2 on p_1 and p_2 be used. The properties of the system (14) with $\varkappa = 0$ and of the system from [1-3] are substantially different. Specifically, in distinction to the latter system, for Eqs. (14) the initial conditions for both pressures p_1 and p_2 may be specified.

If $\varkappa = 0$, then, expressing p_1 from the second equation of (14) through p_2 and substituting the result into the first equation, we obtain

$$(1+s)\frac{\partial p_2}{\partial t}+s\tau\frac{\partial^2 p_2}{\partial t^2}=\gamma\Delta\left(p_2+\tau\frac{\partial p_2}{\partial t}\right). \tag{15}$$

In this case the pressure in the fissures also satisfies such an equation.

Equations (14) coincide in form with the partial variant of the equations of heat conduction in a two-phase medium with nonzero thermal diffusivities in both phases corresponding to neglecting convective heat transfer in the relative motion of the phases (see, e.g., [6]). In the case under examination the part of the mean phase temperatures is taken over by the mean pressures in the fissures and blocks, and the relaxation terms do not describe the equalization of the phase temperatures by interphase heat exchange but the equalization of the mentioned pressures as a result of overflow from the porous blocks into the fissures. In the special case of $\varkappa = 0$, Eqs. (14) coincide with the equation of heat conduction in a disperse medium in which only heat conduction through the continuous phase is considerable. The last equations were investigated and solved for special cases by very many authors. If the characteristic time of change of temperature fields is much longer than the time of internal relaxation τ, then it is permissible to examine instead of the system of equations of heat conductions by phases, the so-called "equivalent" equation for the temperature of the continuous phase; the methods of obtaining it and the conditions of its applicability were discussed in detail in [7] (see also [8]).

These same methods may also be applied in the case under examination. If we express p_2 from the second equation of (14) for $\varkappa = 0$ through p_1 in general operator form and use the formal expansion of the operator into a Taylor series, we obtain

$$p_2 = \frac{1}{1 + \tau\partial/\partial t} p_1 = \sum_{n=0}^{\infty} (-1)^n \tau^n \frac{\partial^n p_1}{\partial t^n}. \tag{16}$$

This operator expansion is completely equivalent to the expansion of the ratio of the Laplace transforms p_1 and p_2, obtained from the second equation of (14) after its Laplace transform, by powers of the transformation variable which may be considered small when $\tau/T \ll 1$, where T is the characteristic time of change of the pressure fields. Different approximations correspond to the maintaining of a different number of first terms in the series of derivatives in (16). In the zeroth approximation $p_1 = p_2$. In this case it is indispensable to neglect the derivative with respect to time in the first equation of (14), too, i.e., this approximation corresponds to the steady-state process of filtration. In the first approximation we obtain the parabolic equation

$$(1 + s)\frac{\partial p_1}{\partial t} = \gamma\Delta p_1, \tag{17}$$

describing the filtering throughflow of the liquid contained both in the fissures and in the blocks. It was shown in [7] that the approximation, effected by Eq. (17) for the initial system (14) or for Eq. (15), is unequally suitable at different points of the region of flow.

The subsequent, second approximation yields an equivalent equation containing the second derivative with respect to time and belonging to the elliptical type:

$$(1 + s)\frac{\partial p_1}{\partial t} - \tau\frac{\partial^2 p_1}{\partial t^2} = \gamma\Delta p_1. \tag{18}$$

The solutions of this equation bring the corresponding solutions of (14) or (15) closer together uniformly along the coordinates with $\tau/T \ll 1$ [7]. The equations of the subsequent approximations contain derivatives with respect to time of higher orders; their solutions may apparently be used for somewhat refining the solutions of Eq. (18) but basically they do not introduce anything new.

The condition that (18) be approximately correct lies in the requirement that $\tau/T \ll 1$. It is very probable that this same condition is also indispensable for the initial systems (13) and (14) to be correct. When relation (9) is written for the mass flow of liquid between the examined continua with the time-independent coefficient α, it is in fact admitted that the flow q is quasisteady. This also means precisely the implicit assumption that the characteristic time of substantial change of the pressures p_1 and p_2 is much longer than the relaxation time τ. It is therefore permissible to use the equivalent equation (18) instead of (14) or (15), apparently without loss of accuracy of the physical model of liquid motion in fissured and porous strata.

For fissures materials ($\varkappa = 0$) we have instead of (13)

$$s\frac{\partial \varphi}{\partial t} = \nabla\cdot\left(\frac{\gamma}{4}\cdot\nabla\varphi^4\right), \quad \varphi = \frac{p_1 - \sigma}{p^\circ - \sigma}. \tag{19}$$

This type of equation was introduced repeatedly in the past in connection with the description of filtering processes of different nature [3, 5]. It is clear that in this case motion is possible only in the range $p_1 > \sigma$.

For the steady-state process we obtain from (13)

$$\nabla\cdot\left[\left(\frac{p_1-\sigma}{p^\circ-\sigma}\right)^3\gamma\cdot\nabla p_1\right]=-\varkappa\Delta p_2=\frac{p_1-p_2}{\tau},\ p_1>\sigma, \tag{20}$$

$$\Delta p_2=0,\ p_1<\sigma.$$

The solutions of these equations on the boundaries $p_1 = \sigma$ of regions with single-phase and two-phase throughflow have to satisfy the conditions of continuity of the full liquid flow and of pressure in porous blocks.

In conclusion we want to point out that the rather great pressure drop at the bottom of boreholes (or galleries) in fissured and porous, and all the more so in fissured strata may have undesirable consequences in the operation of these boreholes. In fact, in this case there originates a region near the borehole in which fissure permeability greatly decreases or vanishes altogether, i.e., a zone appears in which throughflow is effected only via blocks with low permeability. The natural local unloading of the stratum upon piping of the boreholes weakens this effect but in any case it may be expected that the yield of the borehole is not a monotonically decreasing function of the pressure at the bottom, as is the case with the ordinary type of stratum, but that it attains a maximum at a fully determinate (fairly high in the case of deep-lying strata) value of this pressure. On the other hand, the substantially underestimated yields with specified borehole bottom pressure, and also the slower rates of restoring straum pressure when boreholes in fissured and fissure-porous strata are shut down compared with ordinary strata whose permeability and porosity depend only weakly on the state of stress and stratum pressure, all this may lead to considerable errors in estimating the parameters of a stratum and the oil reserves it contains. In fact, if the standard methods of interpreting the curves of pressure recovery etc. are used, with which it is understood that the permeability of a stratum is practically constant, then the investigation data will formally correspond to a considerably reduced value of permeability compared with the real permeability of the stratum not disturbed by boreholes. Obviously, the estimate of the oil reserves in this case will also be too low. To avoid such errors, it is indispensable in the interpretation of field tests to use the solutions of the equations of filtration which explicitly take into account the dependence of permeability both on the state of stress of the stratum and on the pressure in the fissures. This presupposes not only the investigation of such equations in situations that are of applied interest but also, if possible, a more accurate determination of the coefficients of permeability contained in the equations. It follows from the theory of the present work that for this it is necessary to carry out in each actual case a thorough analysis of the state of stress of the stratum and to have at least a minimum of information on the distribution of fissures according to orientation. We emphasize that in addition to the indicated cause of possible underestimation of oil reserves in fissured and porous strata there is also another cause: when the oil contained in porous blocks is not taken into account in very short-term surveys [3].

NOTATION

c, radius of fissures; e_i, unit vectors; E, modulus of elasticity of porous blocks; h, crack opening; k, tensor of permeability; k, permeability; K_p, K_m, coefficients of compressibility determined in (10); l, linear scale of porous blocks; m, porosity; n, unit vector of the normal to the plane of the fissure; p, pressure; q, mass flow density of the liquid from the blocks to the fissures; s, coefficient introduced in (12); T, time scale of the pressure field; t, time; u, filtration rate; α, coefficient in (9); γ, tensor introduced in (12); ε, Poisson ratio of porous blocks; ϰ, piezoconductivity; μ, viscosity; ρ, density; σ, stress tensor; σ, compressive stress; σ^f, first invariant of the tensor of fictitious stresses in porous blocks; τ, relaxation time; φ, function in (19); subscripts 1 and 2, continua modeling a system of fissures and porous blocks, respectively; the degree sign refers to magnitudes determined for some reference value of the parameters.

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MOTION OF A SPHERICAL CLOUD OF BUBBLES IN A LIQUID WITH MOTIONLESS PACKING

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On the basis of Darcy's linear law of resistance, the problem of the ascent of a spherical cloud of bubbles in an infinite liquid with a motionless solid phase is solved. The influence of inertia of the liquid on the character of cloud deformation is discussed.

In investigating the structure of a real bubbling layer with packing under the action of various kinds of perturbation, it is of interest to determine both the distance to which the perturbation of the liquid velocity field excited by a finite region with increased gas content penetrates and the change in this region over time.

In the two-phase case, in the absence of packing, the problem of the collective interaction of bubbles in a cloud was considered in [1], where macroscopic homogeneity of the cloud was assumed, with the consequence that the problem of large-scale liquid motion was not considered, but a new statistical model of the constrained motion of bubbles was proposed. The bubble cloud considered in the present work is macroscopically inhomogeneous, since the gas content is nonuniformly distributed over the liquid-filled space. Therefore, it is necessary to take account of large-scale motion in investigating the hydrodynamic interaction of the phases. In [2], the motion of a macroscopically inhomogeneous cloud of bubbles moving in viscous conditions was considered. The approximate Lamb–Tem method was used in [2]; in this method, in calculating the drag force of the i-th bubble, in the cloud, all the other drag forces are replaced by point forces, when numerical calculation of the combined motion of a few hundred bubbles is possible. However, in the presence of solid phase, no such simple and computationally expedient schematization is possible and, in addition, the bubble motion is usually found to be inertial in character, in practice. On the other hand, if the number of bubbles is sufficiently large, their collective interaction reduces approximately to the interaction of an arbitrarily chosen bubble with the mean velocity field of the liquid arising as a result of the different buoyancies of the elements of the medium, which is uniquely related to the spatial distribution of the gas phase. This approximation may be described by the methods of the mechanics of multivelocity continua based on averaging theory [3]. However, methods of classical filtration theory, which is based on Darcy's linear filtration law, are sufficient for the elucidation of the character of phase motion [4, 5].

A system of phase-continuity and momentum equations is proposed for the description of the phase motion in a three-phase motionless layer; after simple transformations, the system takes the form

$$\partial q/\partial t = \operatorname{div}[(1-q)\bar{V}_1], \tag{1}$$

$$\partial q/\partial t + \operatorname{div}(q\bar{V}_2) = 0, \tag{2}$$

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